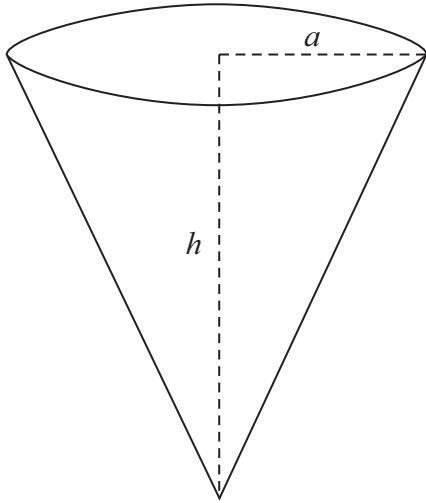




1.

**Figure 1**

A hollow right circular cone, of base radius  $a$  and height  $h$ , is fixed with its axis vertical and vertex downwards, as shown in Figure 1. A particle moves with constant speed  $v$  in

a horizontal circle of radius  $\frac{1}{3}a$  on the smooth inner surface of the cone.

Show that  $v = \sqrt{\left(\frac{1}{3}hg\right)}$ . (7)



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### **Question 1 continued**

Q1

(Total 7 marks)



P 4 2 8 2 9 A 0 3 2 8

2. A particle of mass 4 kg is moving along the horizontal  $x$ -axis under the action of a single force which acts in the positive  $x$ -direction. At time  $t$  seconds the force has magnitude  $\left(1 + 3t^{\frac{1}{2}}\right)$  N.

When  $t = 0$  the particle has speed  $2 \text{ m s}^{-1}$  in the positive  $x$ -direction. Find the work done by the force in the interval  $0 \leq t \leq 4$

(7)



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## **Question 2 continued**

Q2

(Total 7 marks)



P 4 2 8 2 9 A 0 5 2 8

3. A particle  $P$  of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point  $A$ . The particle  $P$  is held at rest at the point  $B$ , which is 1 m vertically below  $A$ , and then released.

- (a) Find the acceleration of  $P$  immediately after it is released from rest.

(4)

The particle comes to instantaneous rest for the first time at the point C.

- (b) Find the distance  $BC$ .

(6)



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### **Question 3 continued**



### **Question 3 continued**

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### **Question 3 continued**

Q3

(Total 10 marks)



4. A particle  $P$  is moving along the positive  $x$ -axis. At time  $t$  seconds,  $t \geq 0$ ,  $P$  is  $x$  metres from the origin  $O$  and is moving away from  $O$  with velocity  $v$  m s $^{-1}$ , where  $v = \frac{4}{(x + 2)}$ . When  $t = 0$ ,  $P$  is at  $O$ . Find

(a) the distance of  $P$  from  $O$  when  $t = 2$

(5)

(b) the magnitude and direction of the acceleration of  $P$  when  $t = 2$

(5)



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## **Question 4 continued**



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## **Question 4 continued**

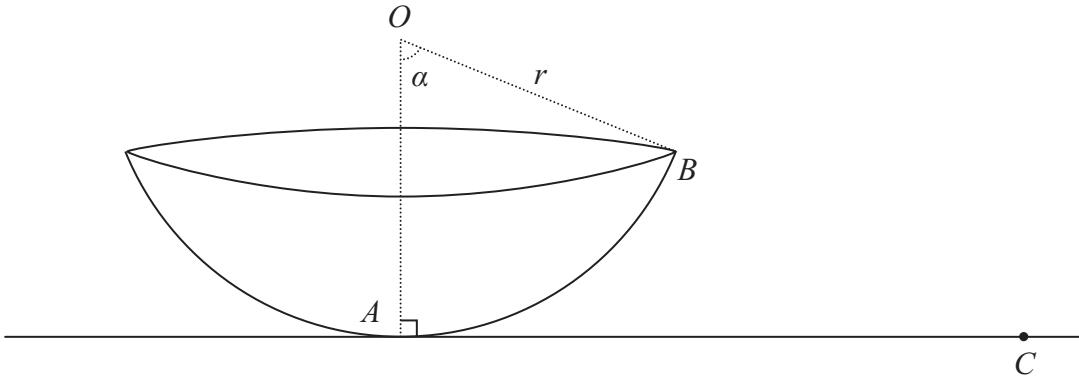
Q4

(Total 10 marks)



P 4 2 8 2 9 A 0 1 3 2 8

5.

**Figure 2**

Part of a hollow spherical shell, centre  $O$  and radius  $r$ , forms a bowl with a plane circular rim. The bowl is fixed to a horizontal surface at  $A$  with the rim uppermost and horizontal.

The point  $A$  is the lowest point of the bowl. The point  $B$ , where  $\angle AOB = \alpha$  and  $\tan \alpha = \frac{3}{4}$ ,

is on the rim of the bowl, as shown in Figure 2. A small smooth marble  $M$  is placed inside the bowl at  $A$ , and given an initial horizontal speed  $\sqrt{(gr)}$ . The motion of  $M$  takes place in the vertical plane  $OAB$ .

- (a) Show that the speed of  $M$  as it reaches  $B$  is  $\sqrt{\left(\frac{3}{5} gr\right)}$ . (4)

After leaving the surface of the bowl at  $B$ ,  $M$  moves freely under gravity and first strikes the horizontal surface at the point  $C$ . Given that  $r = 0.4\text{m}$ ,

- (b) find the distance  $AC$ . (8)



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### **Question 5 continued**



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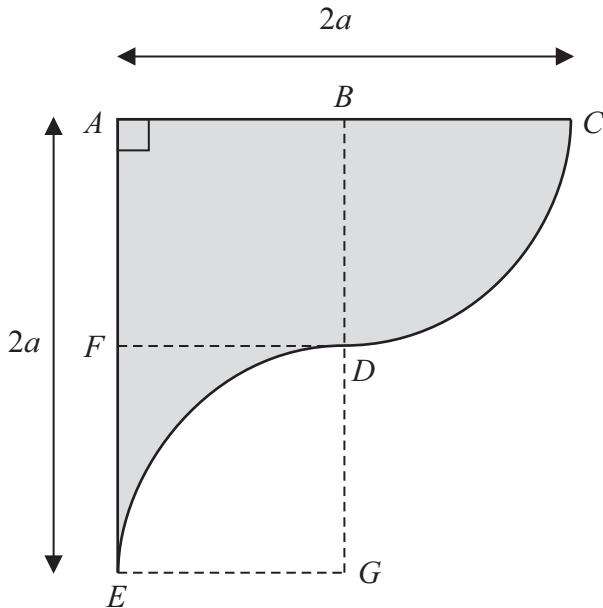
Q5

(Total 12 marks)



P 4 2 8 2 9 A 0 1 7 2 8

6. (a) A uniform lamina is in the shape of a quadrant of a circle of radius  $a$ . Show, by integration, that the centre of mass of the lamina is at a distance of  $\frac{4a}{3\pi}$  from each of its straight edges. (7)



**Figure 3**

A second uniform lamina  $ABCDEF$  is shown shaded in Figure 3. The straight sides  $AC$  and  $AE$  are perpendicular and  $AC = AE = 2a$ . In the figure, the midpoint of  $AC$  is  $B$ , the midpoint of  $AE$  is  $F$ , and  $ABDF$  and  $DGEF$  are squares of side  $a$ .  $BCD$  is a quadrant of a circle with centre  $B$ .  $DGE$  is a quadrant of a circle with centre  $G$ .

- (b) Find the distance of the centre of mass of the lamina from the side  $AE$ . (5)

The lamina is smoothly hinged to a horizontal axis which passes through  $E$  and is perpendicular to the plane of the lamina. The lamina has weight  $W$  newtons. The lamina is held in equilibrium in a vertical plane, with  $A$  vertically above  $E$ , by a horizontal force of magnitude  $X$  newtons applied at  $C$ .

- (c) Find  $X$  in terms of  $W$ . (3)



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## **Question 6 continued**



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### **Question 6 continued**



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## **Question 6 continued**

Q6

**(Total 15 marks)**



P 4 2 8 2 9 A 0 2 3 2 8

7. Two points  $A$  and  $B$  are 4 m apart on a smooth horizontal surface. A light elastic string, of natural length 0.8 m and modulus of elasticity 15 N, has one end attached to the point  $A$ . A light elastic string, of natural length 0.8 m and modulus of elasticity 10 N, has one end attached to the point  $B$ . A particle  $P$  of mass 0.2 kg is attached to the free end of each string. The particle rests in equilibrium on the surface at the point  $C$  on the straight line between  $A$  and  $B$ .

- (a) Show that the length of  $AC$  is 1.76 m.

(4)

The particle  $P$  is now held at the point  $D$  on the line  $AB$  such that  $AD = 2.16$  m. The particle is then released from rest and in the subsequent motion both strings remain taut.

- (b) Show that  $P$  moves with simple harmonic motion.

(4)

- (c) Find the speed of  $P$  as it passes through the point  $C$ .

(2)

- (d) Find the time from the instant when  $P$  is released from  $D$  until the instant when  $P$  is first moving with speed  $2 \text{ m s}^{-1}$ .

(4)

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## **Question 7 continued**



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**Question 7 continued**



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Q7

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**TOTAL FOR PAPER: 75 MARKS**

**END**

